

# Magnetic Field Effect on the Supercurrent of an SNS junction

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In this paper we study the effect of a Zeeman field on the supercurrent of a mesoscopic SNS junction. It is shown that the supercurrent suppression is due to a redistribution of current-carrying states in energy space. A dramatic consequence is that (part of the) the suppressed supercurrent can be recovered with a suitable non-equilibrium distribution of quasiparticles.

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Recently, there has been a revival of interest in the proximity effect [1] which occurs when normal metals (N) are placed in contact with superconductors (S). This is due to the availability of submicron fabrication technology and low temperatures. One of the main lessons learnt in the study of these hybrid structures is that, at sufficiently low temperatures, due to the suppression of inelastic scattering etc., it is essential to properly understand the contribution from individual energies. [2–5]. This applies to, e.g., the conductance between N and S in contact as well as supercurrent in an SNS junction [6–9]

For the latter systems, it is in particular useful to understand the supercurrent being carried at each individual energy, i.e., the current-carrying density of states  $N_J(\epsilon)$ . This quantity is the ordinary density of states weighted by the current that the states carry. The supercurrent is the integral over the energy  $\epsilon$  of the product of  $N_J(\epsilon)$  and the occupation number  $n(\epsilon)$ .

Here we consider again an SNS junction but with an applied magnetic field. It is well-known that magnetic field in general suppresses the supercurrent. This can arise from two completely different mechanisms [10]. First, it can be due to the coupling of the magnetic field to superconductivity via the vector potential. I shall not discuss this effect here. This effect can be suppressed in geometries where the area perpendicular to the magnetic field is sufficiently small. The second mechanism is due to the Zeeman energy. Since pairing is singlet in s-wave superconductors, a physical picture commonly used is that the Zeeman field is a pair-breaking perturbation and hence the Cooper pair amplitude decays in space faster in the presence of the field. In a dirty normal metal with diffusion coefficient  $D$ , this decay length  $\sim \sqrt{D/\hbar}$  [10] for an energy splitting  $h$ . For an SNS junction, the supercurrent thus decreases with field due to a reduction of coupling between the two superconductors.

This, however, is not entirely the full picture. As will be shown below, the main effect of the Zeeman splitting is to shift the current-carrying density of states in energy

space. This is analogous to the behavior of the ordinary density of states under  $h$  [10]. The pairing correlation between the two superconductors remains *long-ranged* at appropriate energies. The supercurrent decreases (*c.f.*, however, below) because of this mentioned shift and the associated change in the occupation of the states (see below for details).

A dramatic consequence of the above is that, under a suitable non-equilibrium distribution of quasiparticles, one can *recover* the suppressed supercurrent. I shall demonstrate this using an experimental arrangement studied in ref [6,7]. A suitable applied voltage can recover (partly) the supercurrent suppressed by a magnetic field.

Consider thus a quasi-one dimensional dirty metal wire (N') of length  $L$  connecting two superconductors. We shall always assume that the junction is in the dirty limit. The supercurrent carrying density of states can be obtained via the angular averaged retarded green's function matrix  $\hat{g}$ . For the present purposes it is sufficient to consider the retarded component and I shall leave out the usual superscript  $R$ .  $\hat{g}$  obeys the normalization condition  $\hat{g}^2 = -\pi^2 \hat{1}$  and the Usadel equation. The latter reads, for position  $x$  within N' ( $0 < x < L$ ) and a magnetic field  $B$  along the  $\hat{z}$  direction,

$$[\epsilon \hat{\tau}_3 + h \hat{\sigma}_3, \hat{g}] + \frac{D}{\pi} \partial_x (\hat{g} \partial_x \hat{g}) = 0 \quad (1)$$

Here  $\epsilon$  is the energy with respect to the Fermi level, and  $h = \mu_e B$ . In order to avoid confusions I shall pretend that electrons have a positive magnetic moment and thus identify the directions of the magnetic moment and spin, with up (down) being the states with lower (higher) Zeeman energy.  $\hat{g}$  at the boundaries  $x = 0$  and  $L$  are given by its corresponding values for the equilibrium superconductors. I shall assume that the magnetic field is perfectly screened in S. In this case the boundary conditions at the superconductors are given by

$$\hat{g} = -\pi \frac{\epsilon \hat{\tau}_3 - \hat{\Delta}}{\sqrt{|\Delta|^2 - \epsilon^2}}. \quad (2)$$

with suitable gap matrices  $\hat{\Delta}$  reflecting the phase difference  $\chi$  between the two superconductors.

Eqn (1) can be simplified by noting that it is block-diagonal, since the pairing is singlet. In the usual  $4 \times 4$  notation, the elements associated with the 1st and 4th rows and columns are decoupled from those of 2nd and 3rd (as already noted in, *e.g.*, ref [11,12]) Moreover, the

matrix equations for these submatrices have the same structure as that in zero field except  $\epsilon \rightarrow \epsilon \pm h$ , corresponding to magnetic moment parallel and antiparallel with the external field. It is then convenient to introduce separately the current-carrying density of states for each spin direction:

$$N_J^\sigma(\epsilon) = \langle \hat{p}_x N^\sigma(\hat{p}, \epsilon, x) \rangle \quad (3)$$

where  $N^\sigma(\hat{p}, \epsilon, x)$  is the density of states for spin  $\sigma$  ( $= \uparrow$  or  $\downarrow$ ),  $\hat{p}$  the direction of momentum. Here I have chosen to label the states with the spin direction of the particles. Note that, *e.g.*, the up-spin particles are associated with the down-spin holes (*c.f.* above). For a given spin  $\sigma$ ,  $N_J^\sigma(\epsilon)$  is related to the appropriate sub-matrix of the green's function  $\hat{g}$  via formulas analogous to those in zero field (see, *e.g.* [8])

The total (number) current density at  $T = 0$  is given by the integration of  $N_J^{\uparrow\downarrow}$  over the occupied (negative) energy states:

$$J_s = v_f \int_{-\infty}^0 d\epsilon [N_J^\uparrow(\epsilon) + N_J^\downarrow(\epsilon)] \quad (4)$$

We shall also introduce  $N_J^{av}(\epsilon) = \frac{1}{2}[N_J^\uparrow(\epsilon) + N_J^\downarrow(\epsilon)]$  as the spin averaged current-carrying density of states.  $J_s$  is related to  $N_J^{av}(\epsilon)$  by

$$J_s = 2v_f \int_{-\infty}^0 d\epsilon [N_J^{av}(\epsilon)], \quad (5)$$

exactly the same formula as in zero field. Here  $v_f$  is the Fermi velocity.

We shall confine ourselves to the case of long junctions ( $E_D \ll |\Delta|$ ), and for definiteness choose  $|\Delta| = 100E_D$ . Here  $E_D \equiv D/L^2$  is the Thouless energy associated with  $N$ . The behavior of the current-carrying density of states  $N_J(\epsilon)$  in zero field has already been studied in detail in [8,9]. I shall only mention some of the more relevant features below.  $N_J(\epsilon)$  vanishes for all energies at  $\chi = 0$ . A typical case for other phase differences  $0 < \chi < \pi$  (we shall always restrict ourselves to this range, the other cases can be obtained by symmetries) is as shown by full line in Fig 1.  $N_J$  is odd in the energy variable  $\epsilon$ . Its major feature consists of a positive peak (labelled by + in Fig 1) at energies of several times the Thouless energy  $E_D$  below the fermi energy, and a corresponding negative peak above (labelled by - in Fig. 1). Also seen are the small undulations as a function of energy for larger energies. This oscillatory behavior is a result of the difference in wave-vectors for the participating particles and holes undergoing Andreev reflection at given  $\epsilon$ . For one dimension and in the clean case the pairing amplitude  $f$  (the off-diagonal elements of  $\hat{g}$  in particle-hole space) oscillates as  $e^{\pm 2i(\epsilon/v_f)x}$ . In the present dirty three dimensional case the same physics results in *e.g.*,  $f \sim \exp \pm [(1-i)\sqrt{\epsilon/D}x]$  for the linearized Usadel equation (*i.e.* the limit of small

pairing amplitudes) At a fixed position, the pairing amplitude and hence the coupling between the two superconductors oscillates as a function of energy.  $N_J(\epsilon)$  also vanishes for  $|\epsilon|$  below a few times  $E_D$ , where the ordinary density of states also vanishes. Both the magnitude of this 'minigap' and the position of peaks mentioned above decrease with increasing phase difference, vanishing as  $\chi \rightarrow \pi$  (where  $N_J$  itself also vanishes for all  $\epsilon$ 's).

The behavior of  $N_J^\sigma(\epsilon)$  under a finite field is also shown in Fig 1, where I have chosen an intermediate  $h$  ( $E_D \ll h \ll |\Delta|$ ) for clarity. As mentioned it is convenient to discuss the current-carrying density of states separately for each spin direction under the presence of  $h$ . For magnetic moment along the applied field, the current carrying density of states is roughly (*c.f.* below) that of zero field except shifted in energy by  $-h$ . *i.e.*  $N_J^\uparrow(\epsilon) \approx N_J^{h=0}(\epsilon + h)$ . Correspondingly  $N_J^\downarrow$  for magnetic moment pointing in the opposite direction is shifted up in energy. At fields  $h$  not too small compared with  $\Delta$ , there is a correction to this picture because, if we assume perfect screening of the magnetic field inside the superconductor as we are doing, the replacement  $\epsilon \rightarrow \epsilon \pm h$  in the Usadel equation does not apply for the boundary condition (2) at  $x = 0$  and  $L$ . This correction is negligible if  $h \ll |\Delta|$  and increases with increasing  $h$ .

In this picture, the reason that the supercurrent at finite  $h$  is suppressed (in general) from that of zero field is *not* pair-breaking, at least for  $h \ll |\Delta|$ . Rather, it is because at finite field  $\sim E_D$ , some of the states that have positive contributions to  $J_s$  were originally occupied at  $h = 0$  but are now empty ( $+\downarrow$ ), whereas some which were originally empty are now occupied ( $-\uparrow$ ) and contribute a negative current.

By the above reasoning, the presence of the magnetic field has a non-trivial effect on the current-phase relationship. An example for a small  $h \ll |\Delta|$  is shown in Fig 2. In zero field  $I(\chi)$  is roughly like a sine function except for a small tilt towards  $\chi \approx \pi$ . When  $h$  increases from zero, one sees that  $I_s$  first starts to decrease for  $\chi$  near  $\pi$  while  $I_s$  at smaller  $\chi$  is unaffected. Only at larger  $h$  would  $I_s$  begin to be suppressed there. This can be readily understood by considering the behavior of  $N_J$  under  $h$  discussed before. Recall that at zero field  $N_J$  has a minigap of order  $E_D$  but  $\chi$  dependent, being smallest when  $\chi$  is near  $\pi$ . Thus when the field  $h$  is increased from 0,  $I_s$  at larger  $\chi$  would be suppressed first since at these  $\chi$ 's, a smaller  $h$  is needed to shift the antipeak  $-\uparrow$  (the peak  $+\downarrow$ ) to below (above) the fermi level.

At higher  $h$  and near  $\chi \approx \pi$  the current also oscillates with  $\chi$ . (See, in particular  $h = 6$  in Fig. 2, where  $I_s$  becomes negative for  $\chi$  slightly less than  $\pi$ , vanishing again at  $\chi = \pi$ ). These features are due to the undulatory structure of  $N_J^\sigma$  as a function of  $\epsilon$  (see Fig 1). At these higher fields the weaker bumps and troughs of  $N_J^\sigma$  (not labelled) cross the fermi level successively. They do so

at fields which are  $\chi$  dependent. Their amplitudes also depends on  $\chi$ .

Since the major effect of the magnetic field is not a suppression of  $N_J^g$  (except for  $h \gtrsim |\Delta|$ ) but a redistribution in energy space, the supercurrent suppression by the magnetic field can, to a certain extent, be *recovered* by a suitable distribution of quasiparticles. Here we consider the case of a ‘controllable Josephson Junction’, studied in [6,7] experimentally and [8,9] theoretically. The device configuration is shown schematically in the inset of Fig 3. The two superconductors S, in general with a phase difference  $\chi$ , are at zero voltage. Equal but opposite voltages  $V$  are applied on the normal (N) reservoirs. The N and S reservoirs are connected by quasi-one dimensional normal wires as shown. We are interested in the effect of the voltage  $V$  on the supercurrent  $I_s$  between the S reservoirs. At  $V = 0$ , the distribution function  $n(\epsilon)$  is of the usual equilibrium form and is given by 1 for  $\epsilon < 0$  and 0 otherwise ( $T = 0$ ), as shown by dotted lines in Fig 3. Under a finite  $V$ ,  $n(\epsilon)$  for  $|\epsilon| < eV$  becomes  $1/2$  [14] (full line in Fig 3), *i.e.* its effect is to transfer half of the quasiparticles for  $-eV < \epsilon < 0$  to the region  $0 < \epsilon < eV$ . In zero field such a non-equilibrium distribution in general leads to a decrease of the supercurrent between the S reservoirs (see [6–9]), since usually this corresponds to decreasing (increasing) the occupation of states which contribute a positive (negative) current [ + (–) in Fig 1] (full line in Fig 4, [15]; see ref [8,9] for the oscillations at higher  $V$ ’s)

The effect of  $V$  on  $I_s$  for finite  $h$  is also shown in Fig 4. At the fields chosen ( $\gg E_D$ )  $I_s$  at zero voltages have essentially decreased to zero (see also below). As claimed, for a given  $h$ , a suitable choice of  $V$  may ‘enhance’ the supercurrent. For the present case where  $E_D \ll h \ll |\Delta|$ , this enhancement is particularly spectacular for  $eV \approx h$ . These features can be understood by examining the spin-averaged current-carrying density of states  $N_J^{qv}(\epsilon)$ , also shown in Fig 3.  $N_J^{qv}(\epsilon)$  is odd in energy. The behavior of  $N_J^{qv}$  follows directly from  $N_J^{\uparrow\downarrow}$  in Fig 1. For a given  $h \neq 0$ , when  $V$  is increased from zero, we have a transfer of the particles as described before from the states near the antipeak just below the fermi level ( due to  $-\uparrow$ ) to the peak above (arising from  $+\downarrow$ ). The supercurrent thus increases. The strongest enhancement of the supercurrent occurs at  $eV \approx h$ , since the region where this transfer occurs just covers the antipeak below the fermi level. For  $E_D \ll h \ll |\Delta|$ , roughly half of the supercurrent at zero voltage can be recovered at  $eV \approx h$ . This is because at this voltage, the contributions from the region  $-eV < \epsilon < eV$  cancel among themselves and we are left with the integral over states with  $\epsilon < -eV$ . The integral of  $N_J^{qv}$  over the energy in this region is roughly half that of zero field. (compare Fig 3 and Fig 1)

The above statements are not quantitatively precise due to: (i)  $N_J^{\uparrow\downarrow}$  are not simple shifts of  $N_J$  in energy and (ii) oscillatory structures of  $N_J^{\uparrow\downarrow}$ , both mentioned before.

For  $V = 0$ , as can be seen in Fig 4, the supercurrent does not simply decay monotonically with increasing  $h$  but rather shows a damped oscillation. Such a behavior has already been pointed out before [11,12], and was explained in terms of the oscillation of the pair-amplitude with  $h$  at a given distance (the separation between the two superconductors). The present work provides a slightly different but closely connected perspective.  $I_s$  oscillates with  $h$  because  $N_J^g$  does so as a function of energy. The field  $h$  shifts  $N_J^g$  in energy space. The undulatory behavior results when regions of alternating signs of  $N_J^g$  shift through the fermi level.

Some of the physics discussed in this paper, such as the effect of  $h$  on the current-phase relationship, is applicable beyond the dirty limit. I shall however defer these to a future study.

There is recently strong interest in the physics of superconductors in contact with a ferromagnetic material. Many papers have simply modelled the ferromagnet F with a Stoner field. *e.g.* [16] (*c.f.*, however [17]) Within this model the Stoner field is formally equivalent to the Zeeman field here. A much discussed topic is the effect of this field  $h$  on the number of conduction channels [16]. This effect is important only when  $h$  is comparable to the fermi energy of N’ and has not been included in the present calculations.

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  - [13] Here  $N_f$  is the ordinary density of states per spin for the normal state,  $l$  the mean free path. In this unit,  $N_J$  is

just  $-Q$  of ref [8] numerically.

- [14] This is only approximately true: see ref [8]. The corrections, studied in ref [8], depend on the details of the geometry of the ‘cross’ and we shall ignore them for simplicity.
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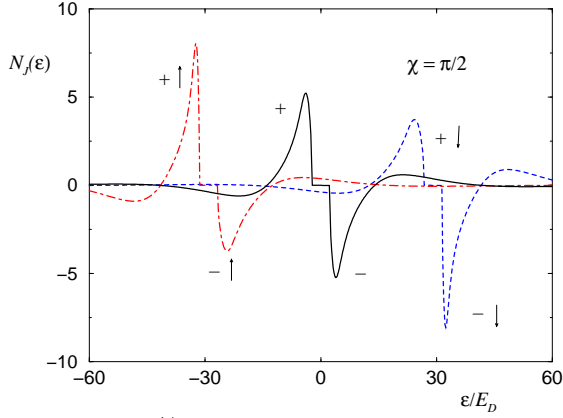


FIG. 1.  $N_J^{\uparrow\downarrow}$  (dot-dashed and dashed respectively and in units of  $N_f l / 6L$  [13]) for a junction with  $\Delta = 100E_D$  at  $h = 30E_D$ . Also shown is  $N_J(\epsilon)$  in zero field (full line).

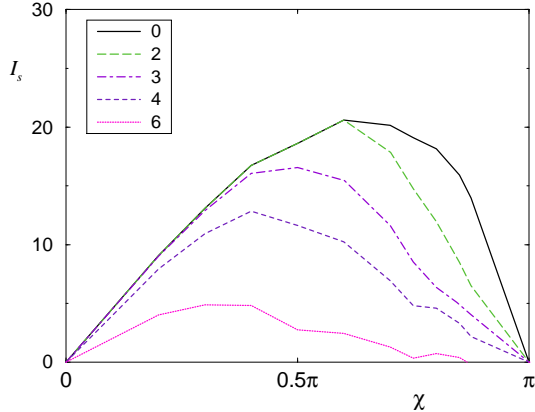


FIG. 2. Current-phase relationships for magnetic fields  $h$  displayed in the legend.  $h$  is in units of  $E_D$ ,  $I_s$  in units of  $E_D / 2R_N$ . Here  $R_N$  is the normal state resistance of  $N'$  ( $\frac{1}{R_N} = 2N_f D S L^{-1}$  with  $S$  the cross-section area of  $N'$ ).

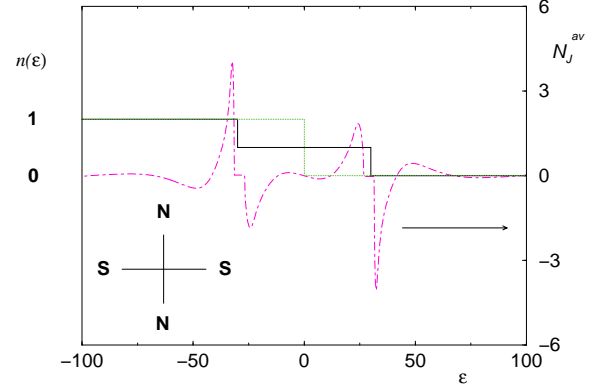


FIG. 3. Explanation of the effect of a voltage on the supercurrent for the device shown in the inset. The occupation numbers  $n(\epsilon)$  (left scale) are shown for  $V = 0$  (dotted) and  $V = 30$  (full line). Also shown is the spin-averaged current-carrying density of states  $N_J^{av}$  (right scale).  $\epsilon$  and  $eV$  are in units of  $E_D$ .

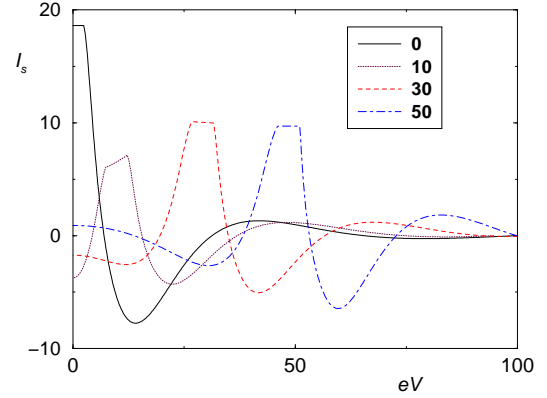


FIG. 4. Supercurrent as a function of voltage  $V$  between the two superconductors  $S$  of the device depicted in the inset of Fig 3. The values of  $h$ , in units of  $E_D$ , are shown in the legend.  $\chi = \pi/2$ .